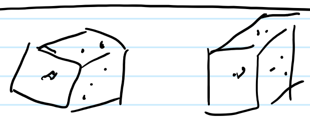
The first meeting

Wednesday, May 5, 2021 2:00 PM

Probability

Integral (Lebesgue)
Measure theory

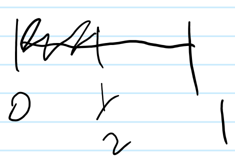


2	$\frac{1}{36}$
3	$\frac{1}{18}$
4	$\vdots$
5	
6	
7	
8	
9	
10	
11	
12	

$$\{2, 3, 4, \dots, 12\}$$
$$\frac{1}{36} \quad \frac{1}{18} \quad \dots \quad \frac{1}{36}$$

$$\sum j \, P\{X = j\} = 7$$

$$P\{X = 2 \text{ or } X = 3\} =$$
$$P(\{X = 2\}) + P(\{X = 3\})$$



$$P\left(X \le \frac{1}{2}\right) = \frac{1}{2} = \frac{\ell([0, \frac{1}{2}])}{\ell([0,1])} =$$

$$A \subset [0,1]$$

$$P(X \in A) = \text{"}\ell\text{"}(A)$$

Banach-Tarski: $\exists\, A, B : A \cup B = [0,1]$,

Re-arrange A and B:

$$A_0 \cup B_0 = [0, 10^6].$$

First target: define a way to measure a size of a set. $\mu(A)$.

X - set.

2^X – subsets of X.
$\mathcal{P}(X)$

$\mathcal{A} \subset 2^X$ – set of subsets of X.

$\mu : \mathcal{A} \to \mathbb{R}_+$.

1) $\mu(X) = 1$.

2) $(A_n)_{n=1}^{\infty}$ $A_n \cap A_m = \emptyset$

$$\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n).$$

$A_n \in \mathcal{A}$.

probability measure

$A \in \mathcal{A}$ $\mu(A)$ – probability that "you are in this set!"

How to define any interesting μ?
What is $\mathcal{A}$?

(X, μ) $f : X \to \mathbb{R}$ $f(X) = \{a_1, \dots, a_n\}$.

$$\int f \, d\mu = a_1 \mu(\{\omega : f(\omega) = a_1\}) + a_2 \mu(\{\omega : f(\omega) = a_2\}) + \dots + a_n \mu(\{\omega : f(\omega) = a_n\})$$

μ – Lebesgue measure on $[0,1]$

$\mu([a,b]) = b - a$ $([a,b] \subset [0,1])$

f_1 – Riemann integrable on $[0,1]$.

$$\int_0^1 f(x)\,dx = \int f \, d\mu.$$